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Perturbative Q^2-power corrections to the structure function g_1

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Abstract. We show that ($\sim 1/(Q^2)^k$) power corrections to the spin structure function g_1 at small x are generated perturbatively from the regulated infrared divergencies. We present the explicit series of such terms as well as the formulae for their resummation. These contributions are not included in the standard analysis of the experimental data. We argue that accounting for such terms can sizably change the impact of the power corrections conventionally attributed to the higher twists.

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1 Introduction

The theoretical description of the Q^2-dependence of the structure function g_1 in perturbative QCD is mostly performed for the kinematic region of large Q^2. However, for the phenomenological analysis of the results of the COMPASS collaboration one needs as far as possible knowledge of g_1 at small x and small Q^2 (see e.g. [1]). Let us recall that the conventional standard approach (SA) based on combining the DGLAP evolution equations [2–5] with phenomenological input for the initial parton densities [6–10] cannot be used for the description of g_1 in this region. Strictly speaking, the SA can be applied only for large values of x ($x \sim 1$) and large Q^2: $Q^2 \gg Q_0^2$, where Q_0^2 is the starting point of the Q^2-evolution. Indeed the small-x region lies beyond the reach of the SA because DGLAP does not include the total resummation of terms $\propto \ln^k 1/x$. In order to describe the experimental data at small x, in the SA one has to include singular terms $\propto x^{-\alpha}$ in the expressions for the initial parton densities. Such factors act as the leading singularities (simple poles) in the Mellin transform of $g_1(x)$ and provide g_1 with Regge asymptotics: $g_1 \sim x^{-\alpha}$ when $x \to 0$.

On the other hand, in our approach [11–13] based on the total resummation of the leading $\ln^k 1/x$ terms in perturbative QCD, the Regge behavior of g_1 at $x \to 0$ appears naturally, independent of the value of Q^2. Indeed it results from the leading singularities of the anomalous dimensions and coefficient functions, which are branching points and not simple poles. The singularities of the anomalous dimensions and coefficient functions coincide. This is very important because it guarantees the independence of Q^2 of the intercepts of g_1. Furthermore by fitting the experimental data in our approach one does not need ad hoc singular factors $\sim x^{-\alpha}$ in the initial parton densities.

The theoretical aspects of the power corrections to the DIS structure functions were considered in [14–19] at large x and Q^2. The interplay between the perturbative and non-perturbative corrections in hard kinematics was recently considered in detail in [20–23]. In fitting experimental data in the small-x region, the discrepancies from the SA predictions are conventionally interpreted (see e.g. [24, 25]) as (non-perturbative) higher twist power ($\sim (1/Q^2)^k$) contributions. However, as the small-x region is beyond the reach of SA, the real size of the higher twist corrections can be erroneously overestimated. It is clear that a systematic account of power corrections $\sim (1/Q^2)^k$ can be satisfactory only when using formulas that already account for the resummation of logarithmic contributions.

We have recently proposed in [26] a generalization of our previous results [11–13] for g_1. Although this extension goes beyond the logarithmic accuracy that we keep, it looks quite natural. In addition to the standard region of large Q^2, this generalization can describe the small-Q^2 region, though in a model-dependent way. Our suggestion, based on the analysis of the Feynman graphs for g_1 at small x, is to replace Q^2 by $Q^2 + \mu^2$ in our previous formulas, with μ being the IR cut-off. Such a shift also leads to the replacement of the variable x by $x' = x + \mu^2/2pq$. The variable x' is similar to the Nachtmann variable.[1]

In the present paper we show that regulating the IR divergencies also is the origin of the power corrections for g_1 at small x. They differ from the well-known power corrections [14–19] related to the resummation of the Sudakov logarithms: first, they come from the ladder Feyn-

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[1] We are grateful to L.N. Lipatov for reminding us of this.

man graphs in the Regge kinematics where virtual gluons are not always soft; then, in contrast to [14–23], we never use[2] the parametrization $\alpha_s = \alpha(Q^2)$ and do not consider the inclusions of power terms [28] into the standard expressions for α_s. We present the explicit formulas for the total resummation of these new corrections and show that at large Q^2 they are $\sim 1/(Q^2)^k$ but in the small-Q^2 region they are $\sim (Q^2)^k$.

The paper is organized as follows: in Sect. 2 we explain why there should be IR perturbative and non-perturbative power corrections to g_1. We also discuss in this section the difference in the IR properties of g_1 at large and small x. In Sect. 3 we recall the essence of the model for g_1 suggested in [26] and list explicit expressions for the singlet and non-singlet g_1 in the kinematic region of small x and arbitrary Q^2. In Sect. 4 we give in more detail than in [26] theoretical arguments in favor of those expressions. The expressions for g_1 enlisted in Sect. 3 and proved in Sect. 4 account for the total resummation of leading logarithms of x and Q^2 when the Q^2 are large, but they are also valid for small Q^2. They implicitly include the power Q^2-corrections. The explicit expressions for the power corrections are extracted from those formulas in Sect. 5. We show here that the power corrections at large and small Q^2 are quite different but the lowest twist contribution to g_1 is always leading regardless of Q^2. Finally, Sect. 6 is for a discussion.

2 Origin of the IR dependent contributions to g_1

It is well known that the DIS structure functions are introduced through the hadronic tensor $W_{\mu\nu}$. In particular, the spin-dependent part of $W_{\mu\nu}$ for the electron–proton DIS is parameterized by the structure functions g_1 and g_2:

$$W_{\mu\nu}^{\rm spin} = \mathrm{i}\epsilon_{\mu\nu\lambda\rho}\frac{Mq_\lambda}{pq}\left[S_\rho\, g_1(x,Q^2) + \left(S_\rho - \frac{Sq}{pq}\right) g_2(x,Q^2)\right], \quad (1)$$

where p, M, S are the proton momentum, mass and spin, respectively, and q is the momentum of the virtual photon. The spin structure functions $g_{1,2}$ have non-singlet, $g_{1,2}^{NS}$, and singlet, $g_{1,2}^{S}$, components. In general, $g_{1,2}$ (and all other DIS structure functions) acquire both perturbative and non-perturbative QCD contributions.

In the first place, there is a non-perturbative term $W_{\mu\nu}^{\rm NPT}$ in the region of small Q^2. For instance, there are known examples of the lattice calculations for some structure functions (see e.g. [29]), although basically $W_{\mu\nu}^{\rm NPT}$ is poorly known.

The other part, $W_{\mu\nu}^{\rm PT}$, of $W_{\mu\nu}$ includes both perturbative and non-perturbative QCD contributions. The standard way to calculate the structure functions is using the factorization. According to this, $W_{\mu\nu}$ is regarded as a convolution,

$$W_{\mu\nu}^{\rm PT} = W_{\mu\nu}^{q} \otimes \Phi_q + W_{\mu\nu}^{g} \otimes \Phi_g\,, \quad (2)$$

of the partonic tensors $W_{\mu\nu}^{q}$ and $W_{\mu\nu}^{g}$ (where q and g label the incoming parton, i.e. a quark or a gluon, respectively), and where we have the probabilities $\Phi_{q,g}$ to find this parton (quark or gluon) in the incoming hadron. The probabilities $\Phi_{q,g}$ include both perturbative and non-perturbative contributions, whereas purely perturbative tensors $W_{\mu\nu}^{q;g}$ describe their x- and Q^2-evolutions. There are no known explicit expressions for $\Phi_{q,g}$. Instead, they are approximated by the initial parton densities δq and δg defined from fitting the experimental data at $Q^2 \sim 1\,\mathrm{GeV}^2$ and $x \sim 1$. The partonic tensors $W_{\mu\nu}^{q}$ and $W_{\mu\nu}^{g}$ evolve these densities into the region where $Q^2 \gg \mu^2$ and $x \ll 1$, with μ^2 being the starting point of the Q^2-evolution. Such a contribution to $W_{\mu\nu}$ is called the lowest twist contribution $W_{\mu\nu}^{\rm LT}$. Besides this, there are the higher twists contributions $W_{\mu\nu}^{\rm HT}$. They can be interpreted either in terms of a more involved convolution or as essentially non-perturbative objects.

In order to calculate $W_{\mu\nu}^{q;g}$, one has to regulate the IR singularities in the Feynman graphs involved. Such a regulation is different for large and small x. At large x one can use DGLAP. In DGLAP this problem is solved assuming a non-zero virtuality μ^2 for the initial partons and imposing the ordering

$$\mu^2 < k_{1\,\perp}^2 < k_{2\,\perp}^2 \ldots < k_{n\,\perp}^2 < Q^2 \quad (3)$$

on the transverse momenta of the ladder partons (the numeration in (3) runs from the bottom to the top of the ladders). Equation (3) makes it manifest that $k_{r\,\perp}$ acts as an IR cut-off for integrating over k_{r+1}. However, the ordering allows one to collect the contributions that are leading at large x only. In order to account for the leading (double-logarithmic) contributions at $x \ll 1$, the upper limit of the integration in (3) should be changed for $(p+q)^2 \approx 2pq$ and the ordering should be lifted. Without the ordering, the IR singularities in the ladder rungs are no longer regulated automatically, so an IR cut-off should be introduced for integration over every loop momentum. This is the reason why the μ-dependence of g_1 is getting more involved at small x. Usually the cut-off is identified with μ, the starting point of the Q^2-evolution, though it is not mandatory. Obviously, the value of μ should be large enough to justify using the perturbative QCD:

$$k_i^2 > \mu^2 > \Lambda_{\rm QCD}^2\,. \quad (4)$$

Generally speaking, there are different ways to introduce IR cut-offs, but providing IR divergent propagators with fictitious masses is most wide-spread. Obviously, no observables should depend on the value of μ and on the ways of its introducing. It means that the explicit μ-dependence in $W_{\mu\nu}^{q;g}$ should be compensated by a μ-dependence of δq and δg. However, the latter are known as phenomenological expressions containing a set of numerical parameters fixed from fitting the experimental data. Those parameters are supposed to be μ-dependent, though in an implicit way.

[2] It is known [27] that these parameterizations fail for g_1 at small x.

Before considering the IR properties of g_1 at small x, let us discuss the well-known expression for the non-singlet g_1 in the standard approach:

$$g_1^{\mathrm{NS}}\left(x, Q^2, \mu^2\right) = \left(e_q^2/2\right) \int_{-\mathrm{i}\infty}^{\mathrm{i}\infty} \frac{\mathrm{d}\omega}{2\pi\mathrm{i}} x^{-\omega} C_q(\omega) \delta q(\omega) \times \exp\left[\int_{\mu^2}^{Q^2} \frac{\mathrm{d}k_\perp^2}{k_\perp^2} \gamma_{qq}\left(k_\perp^2, \alpha_\mathrm{s}(k_\perp^2)\right)\right], \quad (5)$$

where e_q is the electric charge of the quark, C_q is the coefficient function and γ_{qq} is the DGLAP non-singlet anomalous dimension. When γ_{qq} is taken in LO, the integral in (5) is known to be $(1/b)\ln[\ln(Q^2/\mu^2)/\ln(\mu^2/\Lambda^2_{\mathrm{QCD}})]$, with b being the first coefficient of the Gell-Mann–Low function. The expression for the singlet g_1 looks similar, though more involved. Both the coefficient functions and the anomalous dimensions in SA are known in a few first orders in α_s. It means that using (5) is theoretically based for the kinematic region where the x are not far from 1 and $Q^2 \gg \mu^2 \approx 1\ \mathrm{GeV}^2$. The expression for g_1 in (5) depends on the value of μ. This dependence is supposed to disappear when g_1 is complemented by a contribution g_1^{HT} extracted from $W_{\mu\nu}^{\mathrm{HT}}$. In practice, treating of the experimental data on polarized DIS is carried out as follows (see e.g. the recent papers [24, 25] and references therein): the data are compared with g_1 of (5) and the discrepancy is attributed to the impact of the higher twists. However, SA for g_1 is reliable at large x. At the small-x region it should be modified. In the next section we present explicit expressions for g_1^{S} and g_1^{NS} replacing the DGLAP expressions in the small-x region.

3 Expressions for g_1 at small x and arbitrary Q^2

When $x \ll 1$, the contributions $\sim \ln^k(1/x)$ are large, so they should be accounted for to all orders in the QCD coupling. The total resummation of the leading logarithms of x for g_1 was done in [11–13] for the region of $Q^2 \gtrsim \mu^2$. We remind the reader that, contrary to DGLAP, the expressions for g_1 in [11–13] are valid both for large Q^2 and for $Q^2 \sim \mu^2$. Recently, in [26] we have suggested a simple prescription to generalize those results to arbitrary values of Q^2: in the formulas of [11–13] Q^2 should be replaced by $Q^2 + \mu^2$. This conclusion follows from the observation that the contributions of Feynman graphs to g_1 at small x depend on Q^2 through $Q^2 + \mu^2$ only. It automatically leads to the shift $x \to x + z$, with $z = \mu^2/2pq$. As the prescription is beyond the logarithmic accuracy that we kept in our previous papers, we call it a model. The theoretical grounds of this model are given in the next section. According to our results, g_1^{NS}, the non-singlet component of g_1 at the small-x region, is given by the following expression:

$$g_1^{\mathrm{NS}}\left(x+z, Q^2+\mu^2\right) = \left(e_q^2/2\right) \int_{-\mathrm{i}\infty}^{\mathrm{i}\infty} \frac{\mathrm{d}\omega}{2\pi\mathrm{i}} \left(\frac{1}{z+x}\right)^{\omega} C_{\mathrm{NS}}(\omega) \times \delta q(\omega) \left(\frac{Q^2+\mu^2}{\mu^2}\right)^{H_{\mathrm{NS}}(\omega)}, \quad (6)$$

with the new coefficient functions C_{NS},

$$C_{\mathrm{NS}}(\omega) = \frac{\omega}{\omega - H_{\mathrm{NS}}(\omega)} \quad (7)$$

and the anomalous dimensions H_{NS},

$$H_{\mathrm{NS}} = (1/2)\left[\omega - \sqrt{\omega^2 - B(\omega)}\right], \quad (8)$$

where

$$B(\omega) = (4\pi C_\mathrm{F}(1+\omega/2)A(\omega) + D(\omega))/(2\pi^2). \quad (9)$$

$D(\omega)$ and $A(\omega)$ in (9) are expressed in terms of $\rho = \ln(1/x)$, $\eta = \ln(\mu^2/\Lambda^2_{\mathrm{QCD}})$, $b = (33-2n_f)/12\pi$ and the color factors $C_\mathrm{F} = 4/3$, $N = 3$:

$$D(\omega) = \frac{2C_\mathrm{F}}{b^2 N} \int_0^\infty \mathrm{d}\rho \mathrm{e}^{-\omega\rho} \ln\left(\frac{\rho+\eta}{\eta}\right) \times \left[\frac{\rho+\eta}{(\rho+\eta)^2+\pi^2} + \frac{1}{\rho+\eta}\right], \quad (10)$$

$$A(\omega) = \frac{1}{b}\left[\frac{\eta}{\eta^2+\pi^2} - \int_0^\infty \frac{\mathrm{d}\rho \mathrm{e}^{-\omega\rho}}{(\rho+\eta)^2+\pi^2}\right]. \quad (11)$$

H_{NS} and C_{NS} account for the DL and SL contributions to all orders in α_s and, contrary to the DGLAP phenomenology, δq does not contain singular factors. The IR cut-off μ obeys (4). Expression (6) is valid for large Q^2, i.e. for $Q^2 \gg \mu^2$, where $x \gg z$, and for small Q^2, $Q^2 \le \mu^2$, where $x \le z$. The expression for the singlet component, g_1^{S}, of g_1 is more involved:

$$g_1^{\mathrm{S}} = g_1^{(+)} + g_1^{(-)}, \quad (12)$$

with

$$g_1^{(\pm)} = \frac{\langle e_q^2\rangle}{2} \int_{-\mathrm{i}\infty}^{\mathrm{i}\infty} \frac{\mathrm{d}\omega}{2\pi\mathrm{i}} \left(\frac{1}{z+x}\right)^{\omega} \times \left(C_q^{(\pm)}\delta q - \frac{A'}{2\pi\omega^2} C_g^{(\pm)} \delta g\right) \left(\frac{Q^2+\mu^2}{\mu^2}\right)^{\Omega_{(\pm)}}, \quad (13)$$

where $\langle e_q^2\rangle$ stands for the sum of the electric charges: $\langle e_q^2\rangle = 10/9$ for $n_f = 4$, δq and δg are the initial quark and gluon densities.

The exponents $\Omega_{(\pm)}$ and coefficient functions $C_{q,g}^{(\pm)}$ are

$$\Omega_{(\pm)} = \frac{1}{2}\left[H_{qq} + H_{gg} \pm R\right]. \quad (14)$$

$$C_q^{(+)} = \frac{\omega}{RT}\left[(H_{qq} - \Omega_{(-)})(\omega - H_{gg}) + H_{qg}H_{gq} + H_{gq}(\omega - \Omega_{(-)})\right],$$

$$C_q^{(-)} = \frac{\omega}{RT}\left[(\Omega_{(+)} - H_{qq})(\omega - H_{gg}) - H_{qg}H_{gq} + H_{gq}(\Omega_{(+)} - \omega)\right],$$

$$C_g^{(+)} = \frac{\omega}{RT}\big[(H_{gg}-\Omega_{(-)})(\omega-H_{qq}) + H_{qg}H_{gq} + H_{qg}(\omega-\Omega_{(-)})\big]\left(-\frac{A'}{2\pi\omega^2}\right),$$
$$C_g^{(-)} = \frac{\omega}{RT}\big[(\Omega_{(+)}-H_{gg})(\omega-H_{qq}) - H_{qg}H_{gq} + H_{qg}(\Omega_{(+)}-\omega)\big]\left(-\frac{A'}{2\pi\omega^2}\right). \quad (15)$$

Here

$$R = \sqrt{(H_{qq}-H_{gg})^2 + 4H_{qg}H_{gq}}\,,$$
$$T = \omega^2 - \omega(H_{gg}+H_{qq}) + (H_{gg}H_{qq} - H_{gq}H_{qg}) \quad (16)$$

and

$$H_{qq} = \frac{1}{2}\left[\omega + Z + \frac{b_{qq}-b_{gg}}{Z}\right], \quad H_{qg} = \frac{b_{qg}}{Z},$$
$$H_{gg} = \frac{1}{2}\left[\omega + Z - \frac{b_{qq}-b_{gg}}{Z}\right], \quad H_{gq} = \frac{b_{gq}}{Z}, \quad (17)$$

where

$$Z = \frac{1}{\sqrt{2}}\Big\{\big(\omega^2 - 2(b_{qq}+b_{gg})\big) + \Big[\big(\omega^2 - 2(b_{qq}+b_{gg})\big)^2 - 4(b_{qq}-b_{gg})^2 - 16 b_{gq}b_{qg}\Big]^{1/2}\Big\}^{1/2}, \quad (18)$$

with

$$b_{ik} = a_{ik} + V_{ik}\,. \quad (19)$$

The Born contributions a_{ik} are defined as follows:

$$a_{qq} = \frac{A(\omega)C_{\mathrm{F}}}{2\pi}, \quad a_{gq} = -\frac{n_f A'(\omega)}{2\pi},$$
$$a_{qg} = \frac{A'(\omega)C_{\mathrm{F}}}{\pi}, \quad a_{gg} = \frac{4NA(\omega)}{2\pi}. \quad (20)$$

Finally, the non-ladder contributions are

$$V_{ik} = \frac{m_{ik}}{\pi^2}D(\omega)\,, \quad (21)$$

with

$$m_{qq} = \frac{C_{\mathrm{F}}}{2N}, \quad m_{gg} = -2N^2,$$
$$m_{gq} = n_f\frac{N}{2}, \quad m_{qg} = -NC_{\mathrm{F}}\,. \quad (22)$$

The additional factor $\left(-\frac{A'}{2\pi\omega^2}\right)$ in the coefficients $C_g^{(\pm)}$ in (15), with

$$A'(\omega) = \frac{1}{b}\left[\frac{1}{\eta} - \int_0^\infty \rho\frac{\mathrm{d}\rho\, \mathrm{e}^{-\omega\rho}}{(\rho+\eta)^2}\right], \quad (23)$$

is the small-ω estimate for the quark box diagram that dominates in the Born term relating the initial gluons to the electromagnetic current. $A'(\omega)$ is the Mellin representation of the QCD running coupling α_{s} involved in the quark box.

Besides resummation of the leading logarithms, (6) and (12) differ from DGLAP in the parametrization of α_{s}: the DGLAP prescription is $\alpha_{\mathrm{s}} = \alpha_{\mathrm{s}}(Q^2)$, whereas in our approach α_{s} is replaced by A and A', defined in (11) and (23). Such a difference results in a drastic difference in the form of the Q^2-dependence of g_1 between our approach and SA: instead of the factor $(Q^2+\mu^2)/\mu^2$ in (6) and (12), the SA leads to $\ln(Q^2/\Lambda^2_{\mathrm{QCD}})$.

4 Theoretical grounds for the shift $Q^2 \to Q^2+\mu^2$ in (6) and (12)

Both the singlet and non-singlet components of g_1 obey the following Bethe–Salpeter equation, depicted in Fig. 1:

$$g_1 = g_1^{\mathrm{Born}} + \mathrm{i}\int \frac{\mathrm{d}^4k}{(2\pi)^4}(-2\pi\mathrm{i})\delta\left((q+k)^2 - m_q^2\right)\frac{2k_\perp^2}{(k^2-m_q^2)^2}M(p,k)\,, \quad (24)$$

where the δ-function (together with the factor $-2\pi\mathrm{i}$) corresponds to the cut propagator of the uppermost quark with momentum k and mass m_q coupled to the virtual photon lines and $2k_\perp^2$ appear after simplifying the spin structure of the equation. $M(p,k)$ stands for the cut parton ladders and the initial parton densities. In other words, $M(p,k)$ incorporates both the initial parton densities and radiative corrections to them. This object can be called the polarized parton distribution function. In the present paper we will address it simply as the (parton) distribution, skipping the other terms. For the sake of simplicity we have dropped unessential numerical factors ($e_q^2/2$ for f^{NS} and $\langle e_q^2\rangle/2$ for the singlet) in (24). Obviously, M in (24) cannot depend on Q^2.

4.1 Prescription for the IR regularization of M

The IR divergent contributions to M should be regulated. We follow the standard prescription and assign a fictitious mass μ to the gluons in the IR divergent propagators. In particular, in the ladder graphs such propagators are the

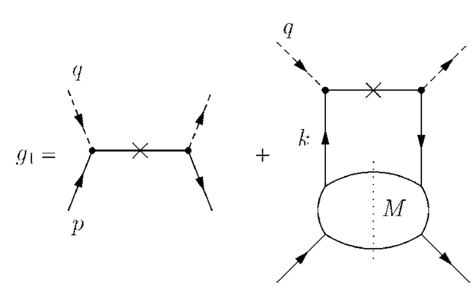

Fig. 1. The Bethe–Salpeter equation for g_1

vertical ones. We assume that the value of μ satisfies (4). In contrast to the gluon ladders, quark ladders are IR stable because the quark mass m_q acts as an IR cut-off. In order to use the same cut-off μ for regulating both gluon and quark IR divergences, we assume that, in addition to (4), $\mu \gg m_q$. After that, m_q can be dropped. Therefore, in order to regulate IR singularities, we should insert μ^2 in the IR divergent, strut (vertical) propagators, for both quarks and gluons. The horizontal propagators (rungs) are IR stable. This converts (24) into

$$g_1 = g_1^{\mathrm{Born}} + \mathrm{i}\int \frac{\mathrm{d}^4 k}{(2\pi)^4}(-2\pi\mathrm{i})\delta((q+k)^2)\frac{2k_\perp^2}{(k^2-\mu^2)^2}M(pk, k^2+\mu^2)\,. \tag{25}$$

4.2 Solving the Bethe–Salpeter equation (25)

As the kinematics $x \ll 1$ is of Regge type, we need to express M in the Regge kinematics as well. Let us notice[3] that the expressions for M can be obtained from our results for g_1 in [11–13] by replacing the external photon virtuality $q^2 = -Q^2$ by the external quark virtuality k^2 and x by $-k^2/w\alpha$. In the first place, we focus on applying (24) to g_1^{NS}, the non-singlet part of g_1 and denote by M^{NS} the involved quark distribution. The expression for M^{NS} accounting for the total resummation of leading logarithmic contributions can also be borrowed from our formula for g_1^{NS} obtained in [12, 13]. This expression reads

$$M^{\mathrm{NS}}(p,k) = \int_{-\mathrm{i}\infty}^{\mathrm{i}\infty}\frac{\mathrm{d}\omega}{2\pi\mathrm{i}}\left(\frac{2pk}{-k^2+\mu^2}\right)^{\omega}\omega f(\omega)\delta q(\omega)\left(\frac{-k^2+\mu^2}{\mu^2}\right)^{H_{\mathrm{NS}}(\omega)}, \tag{26}$$

where $f = 8\pi^2 H_{\mathrm{NS}}$, and $\delta q(\omega)$ is the initial quark density in the ω-space and H_{NS} is given by (8). H_{NS} and f include the total resummation of leading logarithmic contributions. Similarly, the expression for the singlet distribution M^{S} can be obtained from our results for g_1^{S} in [11]. It is convenient to rewrite (25) in terms of the Sudakov variables for momentum k:

$$k = -\alpha q' + \beta p + k_\perp\,, \tag{27}$$

with $q' = q + xp$, so that $q'^2 \approx p^2 \approx 0$. Substituting (26) into (25), using the Sudakov variables and changing the order of the integrations, we obtain the following equation for g_1^{NS}:

$$\begin{aligned} g_1^{\mathrm{NS}} &= g_1^{\mathrm{Born}} + \frac{1}{8\pi^2}\int_{-\mathrm{i}\infty}^{\mathrm{i}\infty}\frac{\mathrm{d}\omega}{2\pi\mathrm{i}}\omega f(\omega)\delta q(\omega) \\ &\times \int \mathrm{d}\alpha\,\mathrm{d}\beta\,\mathrm{d}k_\perp^2 \frac{k_\perp^2}{(w\alpha\beta + k_\perp^2+\mu^2)^2} \\ &\times \delta\left(w\beta + wx\alpha - w\alpha\beta - k_\perp^2 - Q^2\right) \\ &\times \left(\frac{w\alpha}{w\alpha\beta+k_\perp^2+\mu^2}\right)^{\omega}\left(\frac{w\alpha\beta+k_\perp^2+\mu^2}{\mu^2}\right)^{H_{\mathrm{NS}}}, \end{aligned} \tag{28}$$

[3] We will consider such distributions in more detail in our next paper.

where we have denoted $w = 2pq$. As we consider $x \ll 1$, we can neglect $x\alpha$ compared to β. Using the δ-function for integration over β, we arrive at

$$\begin{aligned} g_1^{\mathrm{NS}} &= g_1^{\mathrm{Born}} + \frac{1}{8\pi^2}\int_{-\mathrm{i}\infty}^{\mathrm{i}\infty}\frac{\mathrm{d}\omega}{2\pi\mathrm{i}}\omega f(\omega)\delta q(\omega) \\ &\times \int \frac{\mathrm{d}\alpha\,\mathrm{d}k_\perp^2}{\alpha Q^2 + k_\perp^2+\mu^2}\left(\frac{w\alpha}{\alpha Q^2+k_\perp^2+\mu^2}\right)^{\omega} \\ &\times \left(\frac{\alpha Q^2+k_\perp^2+\mu^2}{\mu^2}\right)^{H_{\mathrm{NS}}}. \end{aligned} \tag{29}$$

The integration region in (25) is shown in Fig. 2. It is outlined by the following restrictions:

$$w \gg k_\perp^2 > 0,\ w \gg w\alpha \gg \alpha Q^2 + k_\perp^2 + \mu^2\,. \tag{30}$$

Integrating over α and $k_\perp^2$ yields different contributions, depending on the ratio between αQ^2 and $k_\perp^2$. The most important contribution comes from the region D in Fig. 2. After integration over α in this region we get

$$\begin{aligned} g_1^{\mathrm{NS}} &= g_1^{\mathrm{Born}} + \frac{1}{8\pi^2}\int_{-\mathrm{i}\infty}^{\mathrm{i}\infty}\frac{\mathrm{d}\omega}{2\pi\mathrm{i}}\omega f(\omega)\delta q(\omega)\frac{1}{\omega} \\ &\times \int_{Q^2}^{w}\frac{\mathrm{d}k_\perp^2}{k_\perp^2+\mu^2}\left(\frac{w}{k_\perp^2+\mu^2}\right)^{\omega}\left(\frac{k_\perp^2+\mu^2}{\mu^2}\right)^{H_{\mathrm{NS}}} \\ &= g_1^{\mathrm{Born}} + \frac{1}{8\pi^2}\int_{-\mathrm{i}\infty}^{\mathrm{i}\infty}\frac{\mathrm{d}\omega}{2\pi\mathrm{i}} f(\omega)\delta q(\omega) \\ &\times \int_{Q^2+\mu^2}^{w+\mu^2}\frac{\mathrm{d}t}{t}\left(\frac{w}{t}\right)^{\omega}\left(\frac{t}{\mu^2}\right)^{H_{\mathrm{NS}}}. \end{aligned} \tag{31}$$

The leading contribution in (31) comes from the lowest limit, $t = Q^2 + \mu^2$, and it gives

$$\begin{aligned} g_1^{\mathrm{NS}} &= g_1^{\mathrm{Born}} + \frac{1}{8\pi^2}\int_{-\mathrm{i}\infty}^{\mathrm{i}\infty}\frac{\mathrm{d}\omega}{2\pi\mathrm{i}}\frac{f(\omega)\delta q(\omega)}{(\omega - H_{\mathrm{NS}})} \\ &\times \left(\frac{w}{(Q^2+\mu^2)}\right)^{\omega}\left(\frac{Q^2+\mu^2}{\mu^2}\right)^{H_{\mathrm{NS}}}, \end{aligned} \tag{32}$$

which proves the validity of the shift $Q^2 \to Q^2 + \mu^2$ suggested in [26]. Let us brush up (32). Replacing $f(\omega)$ by $8\pi^2 H_{\mathrm{NS}}$ we see that in (32)

$$\frac{H_{\mathrm{NS}}}{\omega - H_{\mathrm{NS}}} = -1 + \frac{\omega}{\omega - H_{\mathrm{NS}}}\,. \tag{33}$$

The first term in (33) cancels the Born contribution g_1^{Born}, and the second term is the non-singlet coefficient function

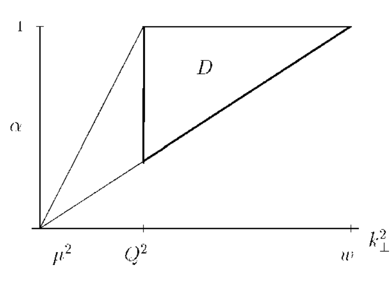

Fig. 2. The integration region over α and $k_\perp^2$ in (29)

(see (8)). Therefore, we arrive at (6) for the non-singlet $g_1^{\rm NS}$ at small x and arbitrary Q^2. Equation (12) for the singlet g_1 in the same kinematic situation can be proved similarly.

5 Infrared power corrections at small x

5.1 Power corrections at large Q^2

In the kinematics where $Q^2 > \mu^2$ and therefore $x > z$, the terms with $Q^2+\mu^2$ in (6) and (12) can be expanded into the following series in μ^2/Q^2:

$$\left(\frac{1}{x+z}\right)^{\omega}\left(\frac{Q^2+\mu^2}{\mu^2}\right)^{H_{\rm NS}} = \left(\frac{1}{x}\right)^{\omega}\left(\frac{Q^2}{\mu^2}\right)^{H_{\rm NS}}\left[1+\sum_{k=1}T_k^{\rm NS}(\omega)\left(\frac{\mu^2}{Q^2}\right)^k\right],$$
$$\left(\frac{1}{x+z}\right)^{\omega}\left(\frac{Q^2+\mu^2}{\mu^2}\right)^{\Omega_\pm} = \left(\frac{1}{x}\right)^{\omega}\left(\frac{Q^2}{\mu^2}\right)^{\Omega_\pm}\left[1+\sum_{k=1}T_k^{(\pm)}(\omega)\left(\frac{\mu^2}{Q^2}\right)^k\right], \quad (34)$$

where

$$T_k^{\rm NS} = \frac{(-\omega+H_{\rm NS})(-\omega+H_{\rm NS}-1)\dots(-\omega+H_{\rm NS}-k+1)}{k!},$$
$$T_k^{(\pm)} = \frac{(-\omega+\Omega_\pm)(-\omega+\Omega_\pm-1)\dots(-\omega+\Omega_\pm-k+1)}{k!}. \quad (35)$$

It allows one to rewrite (6) and (12) as follows:

$$g_1^{\rm NS}(x+z,Q^2) = \tilde{g}_1^{\rm NS}(x,Q^2)+\tilde{g}_1^{\rm NS}(x/y,Q^2)\otimes\sum_{k=1}(\mu^2/Q^2)^k E_k^{\rm NS}(y),$$
$$g_1^{\rm S}(x+z,Q^2) = \tilde{g}_1^{\rm S}(x,Q^2)+\sum_{k=1}(\mu^2/Q^2)^k \times\left[\tilde{g}_1^{(+)}(x/y,Q^2)\otimes E_k^{(+)}(y)+\tilde{g}_1^{(-)}(x/y,Q^2)\otimes E_k^{(-)}(y)\right], \quad (36)$$

where, using the conventional terms, $\tilde{g}_1^{\rm NS}$ and $\tilde{g}_1^{\rm S}$ can be named the non-singlet and singlet components of the lowest twist contribution to g_1:

$$\tilde{g}_1^{\rm NS} = (e_q^2/2)\int_{-\rm i\infty}^{\rm i\infty}\frac{{\rm d}\omega}{2\pi{\rm i}}\left(\frac{1}{x}\right)^{\omega}C_{\rm NS}(\omega)\delta q(\omega)\left(\frac{Q^2}{\mu^2}\right)^{H_{\rm NS}(\omega)},$$
$$\tilde{g}_1{}^S = \tilde{g}_1^{(+)}+\tilde{g}_1^{(-)} = \frac{\langle e_q^2\rangle}{2}\int_{-\rm i\infty}^{\rm i\infty}\frac{{\rm d}\omega}{2\pi{\rm i}}\left(\frac{1}{x}\right)^{\omega} \times\left[\left(C_q^{(+)}\left(\frac{Q^2}{\mu^2}\right)^{\Omega_{(+)}}+C_q^{(-)}\left(\frac{Q^2}{\mu^2}\right)^{\Omega_{(-)}}\right)\delta q -\frac{A'}{2\pi\omega^2}\left(C_g^{(+)}\left(\frac{Q^2}{\mu^2}\right)^{\Omega_{(+)}}+C_g^{(-)}\left(\frac{Q^2}{\mu^2}\right)^{\Omega_{(-)}}\right)\delta g\right] \quad (37)$$

and

$$E_k^{\rm NS}(x) = \int_{-\rm i\infty}^{\rm i\infty}\frac{{\rm d}\omega}{2\pi{\rm i}}\left(\frac{1}{x}\right)^{\omega}T_k^{\rm NS}(\omega),$$
$$E_k^{\pm}(x) = \int_{-\rm i\infty}^{\rm i\infty}\frac{{\rm d}\omega}{2\pi{\rm i}}\left(\frac{1}{x}\right)^{\omega}T_k^{\pm}(\omega). \quad (38)$$

The right-hand sides in (36) are the products of the power corrections and $\tilde{g}_1$. We call these corrections the IR power corrections. The functions $\tilde{g}_1^{\rm S}$, $\tilde{g}_1^{\rm NS}$ in (37) were obtained in [11–13], and they correspond to the lowest twist contribution. They differ from the lowest twist DGLAP expressions for g_1 by the total resummation of the leading logarithms of x and by the new parametrization of $\alpha_{\rm s}$ given by (11) and (23). At small x they include the most important contributions of the LO and NLO DGLAP formulas. When the lowest twist expressions of (37) are used for analysis of the experimental data of the polarized DIS, the power series in the r.h.s. of (36) look as new independent contributions. However, the left-hand sides of (36) account for the total resummation of these corrections. Finally, let us notice that the IR power corrections in (36) have nothing to do with the standard parametrization $\alpha_{\rm s} = \alpha_{\rm s}(Q^2)$ as we do not use it.

5.2 Power corrections at small Q^2

When $Q^2 < \mu^2$, $g_1^{\rm NS}$ and $g_1^{\rm S}$ cannot be expanded similar to (36). The power corrections for small Q^2 are different. Indeed in this case

$$\left(\frac{1}{x+z}\right)^{\omega}\left(\frac{Q^2+\mu^2}{\mu^2}\right)^{H_{\rm NS}} = \left(\frac{1}{z}\right)^{\omega}\left[1+\sum_{k=1}T_k^{\rm NS}(\omega)\left(\frac{Q^2}{\mu^2}\right)^k\right],$$
$$\left(\frac{1}{x+z}\right)^{\omega}\left(\frac{Q^2+\mu^2}{\mu^2}\right)^{\Omega_\pm} = \left(\frac{1}{z}\right)^{\omega}\left[1+\sum_{k=1}T_k^{(\pm)}(\omega)\left(\frac{Q^2}{\mu^2}\right)^k\right]. \quad (39)$$

It leads to the following expressions for g_1 at small Q^2:

$$g_1^{\rm NS}(x+z,Q^2) = \tilde{g}_1^{\rm NS}(z,\mu^2)+\tilde{g}_1^{\rm NS}(z/y,\mu^2)\otimes\sum_{k=1}(Q^2/\mu^2)^k\, E_k^{\rm NS}(y),$$
$$g_1{}^S(x+z,Q^2) = \tilde{g}_1^S(z,\mu^2)+\sum_{k=1}(Q^2/\mu^2)^k \times\left[\tilde{g}_1^{(+)}(z/y,\mu^2)\otimes E_k^{(+)}(y)+\tilde{g}_1^{(-)}(z/y,\mu^2)\otimes E_k^{(-)}(y)\right], \quad (40)$$

where the lowest twist contributions $\tilde{g}_1^{\rm NS}$, $\tilde{g}_1^S$ are given by (37), and $E_k^{\rm NS}$ and E_k^S are defined in (38). Both $\tilde{g}_1^{\rm NS}$ and $\tilde{g}_1^S$ do not depend on x and Q^2. Instead, they depend on the total energy $(p+q)^2$ of the process and are constants when the $2pq$ is fixed. Both the x- and Q^2-dependence are now associated with the power corrections. Equations (36) and (40) show that the IR Q^2-corrections are different for large and small Q^2, so that the series of (40) cannot be extrapolated into the region of small Q^2 and similarly the series in (36) cannot be extrapolated into the large-Q^2 region. Besides the terms with $(Q^2)^k$ given by (40), similar contributions can come from other sources that are beyond our control. However, the $(Q^2)^k$ terms in (40) are multiplied by the functions $\tilde{g}_1^{\rm NS}$ and $\tilde{g}_1^{\rm S}$, which include the total resummation of the leading logarithms, and therefore they are supposed to dominate, at small x, over the coefficients at the other $(Q^2)^k$ terms.

Although the large-Q^2 expansion (34) and the small-Q^2 expansion (39) look quite similar, the power corrections to $g_1^{\rm NS}$ are actually different for large and small Q^2. It is easy to check that the term linear in μ^2/Q^2 is present in the large-Q^2 expansion of (36) for $g_1^{\rm NS}$, while the term with Q^2/μ^2 is absent in the $g_1^{\rm NS}$ expansion of (40).

6 Discussion

At the region of small x, the DGLAP ordering (3) should be lifted for accounting for leading logarithms of $1/x$, so the IR cut-off μ should be introduced explicitly in order to regulate the IR divergencies in every rung of the Feynman graphs contributing to g_1. Similarly to DGLAP, μ can also play the role of the starting point of the Q^2-evolution, though not obligatory. With both the perturbative and non-perturbative contributions accounted for, g_1 does not depend on μ. However, this dependence does exist when the non-perturbative contributions are either neglected or accounted for implicitly through the fits for the initial parton densities. In this case the structure function g_1 depends on the value of μ and on the way it has been introduced. Introducing μ as the fictitious mass inserted into the IR divergent propagators leads to (6) and (12) suggested in [26] for g_1. The expressions of (6) and (12) include the total resummation of double-logarithms and the most important part of the single-logarithms of x. They are obtained from our previous results with the shift $Q^2 \to Q^2 + \mu^2$. The theoretical grounds for such a shift are given by (9)–(32). Equations (6) and (12), in contrast to DGLAP, can be used both for large and small Q^2. Having been expanded into the series in $1/(Q^2)^k$ at large Q^2 (or into the series in Q^2 at small Q^2), (6) and (12) yield the power Q^2-corrections. The series of the corrections are represented by expressions (36) and (40). The power series of (36) and (40) for large and small Q^2 are derived from the same formulas. However, after the expansion has been made, they cannot be related to each other with simply varying Q^2. We suggest that accounting for the new source of the power contributions given by (6) and (12) can sizably change the conventional analysis of the higher twists contributions to the polarized DIS, because such contributions appear in the present analysis of the experimental data as a discrepancy between the experimental data and the SA predictions. Let us recall that in [30] we showed that the singular ($\sim x^{-\alpha}$) factors in the standard fits mimic the total resummations of $\ln^k x$, i.e. they have a purely perturbative origin, contrary to previous common expectations. Similarly, a good portion of the commonly believed non-perturbative power corrections in the conventional analysis of the experimental data can actually be of perturbative IR origin. However, being misinterpreted as non-perturbative terms, they can mimic the power expansion in (36). In particular, (6) predicts that the power $\sim 1(Q^2)^k$-corrections to $g_1^{\rm NS}$ should appear at $Q^2 \gtrsim 1\ {\rm GeV}^2$ and cannot appear at smaller values of Q^2. It agrees with the phenomenological observations obtained in [24, 25] from conventional analysis of experimental data. On the other hand, (12) predicts that similar power corrections to the singlet g_1 should be seen at greater values of Q^2. Clearly, the use of (6) and (12) for the lower twist contributions to g_1, instead of DGLAP, would allow one to revise the impact of the genuine higher twists contributions, which are known to be of non-perturbative origin.

Finally, we would like to remind the reader that our results explicitly depend on the IR cut-off μ. As pointed out in Sect. 2, such a dependence would vanish if analytic expressions for the probabilities $\Phi_{q,g}$ were obtained and used in (6) and (12) instead of δq and δg.

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